THE "MATHEMATICS" OF RELATIVITY

by

Ardeshir Mehta

Sunday, November 4, 2001

ABSTRACT

The Galilean theorem of addition of velocities is proved hereunder as a mathematical theorem. Since all Relativistic formulae are derived from the postulate of the constancy of the speed of light, which contradicts this Galilean theorem, and since in mathematics no axiom, postulate, proposition or theorem may contradict any other, it is proved hereunder that the postulate of the constancy of the speed of light cannot logically form a part of mathematics as we know it.

INTRODUCTION

The "mathematics" — including the "geometry" — of Relativity are based, not *only* on the axioms of mathematics (such as those of Peano, or those enunciated by Zermelo and Fraenkel, later extended by John von Neumann) and on the postulates and propositions of geometry, Euclidean or otherwise,[1] but *also* on the postulate that light propagates in a vacuum at a speed which is constant for all observers, regardless of the speed of the observer relative to the source of the light.

Thus for their formulation, the "mathematics" and "geometry" of Relativity require a postulate *additional* to the axioms, postulates and propositions from which the *rest* of mathematics and geometry (as we know them to be) are formulated.

However, logically speaking, no axiom, postulate or proposition in mathematics and geometry may contradict another; nor may it — nor any theorem derived from it — contradict any other theorem. If this occurs, that particular axiom, postulate or proposition cannot logically be a part of mathematics and/or geometry.

We logically and mathematically *prove* hereunder the Galilean theorem of addition of velocities. Since the so-called Relativistic "theorem" of addition of velocities contradicts the Galilean theorem, it is demonstrated logically that the postulate on which Relativistic "mathematics" and "geometry" are based — namely the postulate of the constancy of the speed of light — cannot be a part of mathematics and/or geometry as we know them.

---

[1] See for example <http://homepage.mac.com/ardeshir/MathAxioms&GeomPostulates.html>.

PROOF

Let there be an inertial frame of reference **F** in which there is an observer **O** possessing a clock **C** for measuring time, as well as other instruments — such as rods — for measuring distances. Let two point-like bodies $\mathbf{B_1}$ and $\mathbf{B_2}$ be moving rectilinearly and uniformly past the observer **O** in opposite directions, at their closest point each body passing at a negligible distance from **O** and from the other body. Let both $\mathbf{B_1}$ and $\mathbf{B_2}$ pass **O** at a single time instant $\mathbf{t_0}$ as indicated by the clock **C**, the body $\mathbf{B_1}$ moving at a velocity $\mathbf{v_1}$ relative to **O**, and the body $\mathbf{B_2}$ moving at a velocity $\mathbf{v_2}$ relative to **O**.

Let the following be defined:

I $\mathbf{t_X}$: any given time instant, as indicated by the clock **C**, after $\mathbf{t_0}$;

II **T** : the time interval between $\mathbf{t_X}$ and $\mathbf{t_0}$, as indicated by the clock **C**;

III $\mathbf{d_1}$: the distance between **O** and $\mathbf{B_1}$, as measured in the frame **F** of the observer **O**, at time instant $\mathbf{t_X}$;

IV $\mathbf{d_2}$: the distance between **O** and $\mathbf{B_2}$, as measured in the frame **F** of the observer **O**, at time instant $\mathbf{t_X}$;

V **D** : the distance between $\mathbf{B_1}$ and $\mathbf{B_2}$, as measured in the frame **F** of the observer **O**, at time instant $\mathbf{t_X}$;

VI **V** : the relative velocity between $\mathbf{B_1}$ and $\mathbf{B_2}$, as measured in the frame **F** of the observer **O**; and

VII **relative velocity** : change in distance $\Delta\mathbf{d}$ between any two bodies divided by the time interval $\Delta\mathbf{t}$ required to effect the change $\Delta\mathbf{d}$, as measured by any *single* observer.

Then at time instant $\mathbf{t_X}$ as indicated by the clock **C**:

1. $\mathbf{d_1 = v_1T}$,

2. $\mathbf{d_2 = v_2T}$; and

3. $\mathbf{D = (d_1 + d_2) = (v_1T + v_2T)}$.

4. So in the frame **F** of the observer **O**, $\mathbf{V = D/T}$
$\mathbf{= (d_1 + d_2)/T}$
$\mathbf{= (v_1T + v_2T)/T}$
$\mathbf{= (v_1T)/T + (v_2T)/T}$
$\mathbf{= (v_1 + v_2)}$.

This logically and mathematically *proves* the Galilean theorem of addition of velocities.[2]

[2] Note that in the above calculation, there is no restriction whatsoever placed on the *magnitude* of the velocities $\mathbf{v_1}$ and $\mathbf{v_2}$. Thus they can even be so-called "Relativistic" velocities — *i.e.,* velocities approaching that of light.

5. Now the Lorentz transformation equations and the geometry of Minkowski space-time are obtained using the Relativistic postulate of the constancy of the speed of light regardless of the speed of the source of the light or of its observer; and among the equations calculated using this postulate is the so-called Relativistic "theorem" of "addition" (or more accurately, "compounding") of velocities, *viz.,* $\mathbf{V = (v_1 + v_2)/(1 + v_1v_2/c^2)}$, where $\mathbf{c}$ is the speed of light in a vacuum.

6. But the equation in 4. above contradicts the equation in 5. above.[3]

7. Since the equation in 4. above has been mathematically logically *proven*, and since in mathematics and logic, no theorem may contradict any other, the equation in 5. above cannot be a mathematical or logical theorem: *i.e.,* a formula or statement which can be *logically and mathematically proven;* and as a corollary, the additional postulate which is required to formulate the equation in 5. above — namely the postulate of the constancy of the speed of light — cannot be a valid postulate of mathematics, geometry and/or logic as we know them.

Q.E.D.

Comments? E-mail me.

[3] If it were correct that $\mathbf{V = (v_1 + v_2)/(1 + v_1v_2/c^2)}$, which is less than $\mathbf{V = (v_1 + v_2)}$, then the distance between $\mathbf{B_1}$ and $\mathbf{B_2}$, namely $\mathbf{D = VT = [(v_1 + v_2)/(1 + v_1v_2/c^2)]T}$, would be less than $\mathbf{D = VT = (v_1 + v_2)T = [(v_1T) + (v_2T)] = (d_1 + d_2)}$ — or in other words, the distance, as measured by the observer **O**, between $\mathbf{B_1}$ and $\mathbf{B_2}$ would be less than the sum of the distances between $\mathbf{B_1}$ and **O** on the one hand, and **O** and $\mathbf{B_2}$ on the other, *also* as measured by the observer **O** ... which is impossible