THE AXIOMS OF MATHEMATICS
and
THE POSTULATES AND PROPOSITIONS OF EUCLIDEAN GEOMETRY

DEFINITIONS:

Theorem: a formula in mathematics (or symbolic logic) which is proven from other formulae, or from a given set of axioms, propositions and postulates (which are statements accepted as true without proof).

Proof: a finite series of formulae, of which each is either an axiom, proposition or postulate, or an immediate consequence of two previous ones.

Immediate Consequence: a formula **c** is called an "immediate consequence" of **a** and **b** — where **a**, **b** and **c** are each either a formula, an axiom, a proposition and/or a postulate — if **a** is the formula **(~(b))∨(c)**.

PEANO'S AXIOMS:

First Axiom: **1** is a natural number.[1]

Second Axiom: For each **x** there exists exactly one natural number, called the successor of **x**, which will be denoted by fx.

Third Axiom: There exists no number whose successor is **1**.

Fourth Axiom: If $\boldsymbol{f}$**x** = $\boldsymbol{f}$**y**, then **x** = **y**. That is, for any given number there exists either no number or exactly one number whose successor is the given number.

Fifth Axiom (Axiom of Induction):

Let there be given a set **M** of natural numbers, with the following properties:
I. **1** belongs to **M**.
II. If **x** belongs to **M** then so does $\boldsymbol{f}$**x**.

Then **M** contains all the natural numbers.

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[1] Sometimes **0** is used in place of **1**. But in the present case this is irrelevant.

EUCLID'S POSTULATES:[2]

First Postulate: A straight line can be drawn from any point to any point.

Second Postulate: A finite straight line can be produced continuously in a straight line.

Third Postulate: A circle can be described with any centre and radius.

Fourth Postulate: All right angles equal one another.

Fifth Postulate: If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

EUCLID'S PROPOSITIONS:[2]

Proposition 1: An equilateral triangle can be constructed on a given finite straight line.

Proposition 2: A straight line equal to a given straight line can be placed with one end at a given point.

Proposition 3: From the greater of two given unequal straight lines a straight line equal to the less can be cut off.

Proposition 4: If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.

Proposition 5: In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.

Proposition 6: If in a triangle two angles equal one another, then the sides opposite the equal angles also equal one another.

Proposition 7: Given two straight lines constructed from the ends of a straight line and meeting in a point, there cannot be constructed from the ends of the same straight line, and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each equal to that from the same end.

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[2] From the Web page <http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html#posts>.

Proposition 8: If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.

Proposition 9: A given rectilinear angle can be bisected.

Proposition 10: A given finite straight line can be bisected.

Proposition 11: A straight line can be drawn at right angles to a given straight line from a given point on it.

Proposition 12: A straight line perpendicular to a given infinite straight line can be drawn from a given point not on it.

Proposition 13: If a straight line stands on a straight line, then it makes either two right angles or angles whose sum equals two right angles.

Proposition 14: If with any straight line, and at a point on it, two straight lines not lying on the same side make the sum of the adjacent angles equal to two right angles, then the two straight lines are in a straight line with one another.

Proposition 15: If two straight lines cut one another, then they make the vertical angles equal to one another.
Corollary: If two straight lines cut one another, then they will make the angles at the point of section equal to four right angles.

Proposition 16: In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles.

Proposition 17: In any triangle the sum of any two angles is less than two right angles.

Proposition 18: In any triangle the angle opposite the greater side is greater.

Proposition 19: In any triangle the side opposite the greater angle is greater.

Proposition 20: In any triangle the sum of any two sides is greater than the remaining one.

Proposition 21: If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

Proposition 22: A triangle can be constructed out of three straight lines which equal three given straight lines: thus it is necessary that the sum of any two of the straight lines should be greater than the remaining one.

Proposition 23: A rectilinear angle can be constructed equal to a given rectilinear angle on a given straight line and at a point on it.

Proposition 24: If two triangles have two sides equal to two sides respectively, but have one of the angles contained by the equal straight lines greater than the other, then they also have the base greater than the base.

Proposition 25: If two triangles have two sides equal to two sides respectively, but have the base greater than the base, then they also have the one of the angles contained by the equal straight lines greater than the other.

Proposition 26: If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.

Proposition 27: If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.

Proposition 28: If a straight line falling on two straight lines makes the exterior angle equal to the interior and opposite angle on the same side, or the sum of the interior angles on the same side equal to two right angles, then the straight lines are parallel to one another.

Proposition 29: A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.

Proposition 30: Straight lines parallel to the same straight line are also parallel to one another.

Proposition 31: A straight line can be drawn through a given point parallel to a given straight line.

Proposition 32: In any triangle, if one of the sides is produced, then the exterior angle equals the sum of the two interior and opposite angles, and the sum of the three interior angles of the triangle equals two right angles.

Proposition 33: Straight lines which join the ends of equal and parallel straight lines in the same directions are themselves equal and parallel.

Proposition 34: In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.

Proposition 35: Parallelograms which are on the same base and in the same parallels equal one another.

Proposition 36: Parallelograms which are on equal bases and in the same parallels equal one another.

Proposition 37: Triangles which are on the same base and in the same parallels equal one another.

Proposition 38: Triangles which are on equal bases and in the same parallels equal one another.

Proposition 39: Equal triangles which are on the same base and on the same side are also in the same parallels.

Proposition 40: Equal triangles which are on equal bases and on the same side are also in the same parallels.

Proposition 41: If a parallelogram has the same base with a triangle and is in the same parallels, then the parallelogram is double the triangle.

Proposition 42: A parallelogram can be constructed equal to a given triangle in a given rectilinear angle.

Proposition 43: In any parallelogram the complements of the parallelograms about the diameter equal one another.

Proposition 44: To a given straight line in a given rectilinear angle, a parallelogram equal to a given triangle can be applied.

Proposition 45: A parallelogram can be constructed equal to a given rectilinear figure in a given rectilinear angle.

Proposition 46: A square can be described on a given straight line.

Proposition 47: In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

Proposition 48: If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is right.